

Turbulent Flows
 Stephen B. Pope
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Solution to Exercise 10.12

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In homogeneous turbulence, the k and ε equations become

$$\frac{dk}{dt} = \mathcal{P} - \varepsilon, \quad (1)$$

and

$$\frac{d\varepsilon}{dt} = C_{\varepsilon 1} \frac{\mathcal{P}\varepsilon}{k} - C_{\varepsilon 2} \frac{\varepsilon^2}{k}. \quad (2)$$

Consider the quantity Z defined by

$$Z = C_Z k^p \varepsilon^q. \quad (3)$$

Differentiating Eq. 3 with respect to t , we get

$$\frac{dZ}{dt} = C_Z p k^{p-1} \varepsilon^q \frac{dk}{dt} + C_Z k^p q \varepsilon^{q-1} \frac{d\varepsilon}{dt}. \quad (4)$$

Substituting Eqs.1 and 2 into 4, we get

$$\begin{aligned} \frac{dZ}{dt} &= C_Z p k^{p-1} \varepsilon^q (\mathcal{P} - \varepsilon) + C_Z k^p q \varepsilon^{q-1} \left(C_{\varepsilon 1} \frac{\mathcal{P}\varepsilon}{k} - C_{\varepsilon 2} \frac{\varepsilon^2}{k} \right) \\ &= \frac{Zp}{k} (\mathcal{P} - \varepsilon) + \frac{Zq}{\varepsilon} \left(C_{\varepsilon 1} \frac{\mathcal{P}\varepsilon}{k} - C_{\varepsilon 2} \frac{\varepsilon^2}{k} \right) \\ &= C_{Z1} \frac{Z\mathcal{P}}{k} - C_{Z2} \frac{Z\varepsilon}{k}, \end{aligned} \quad (5)$$

where

$$C_{Z1} = p + qC_{\varepsilon 1} \quad (6)$$

$$C_{Z2} = p + qC_{\varepsilon 2}. \quad (7)$$

The entries in Table 10.2 are obtained by substituting $C_{\varepsilon 1} = 1.44$, $C_{\varepsilon 2} = 1.92$ and the given values of p and q into Eqs. 6 and 7.

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